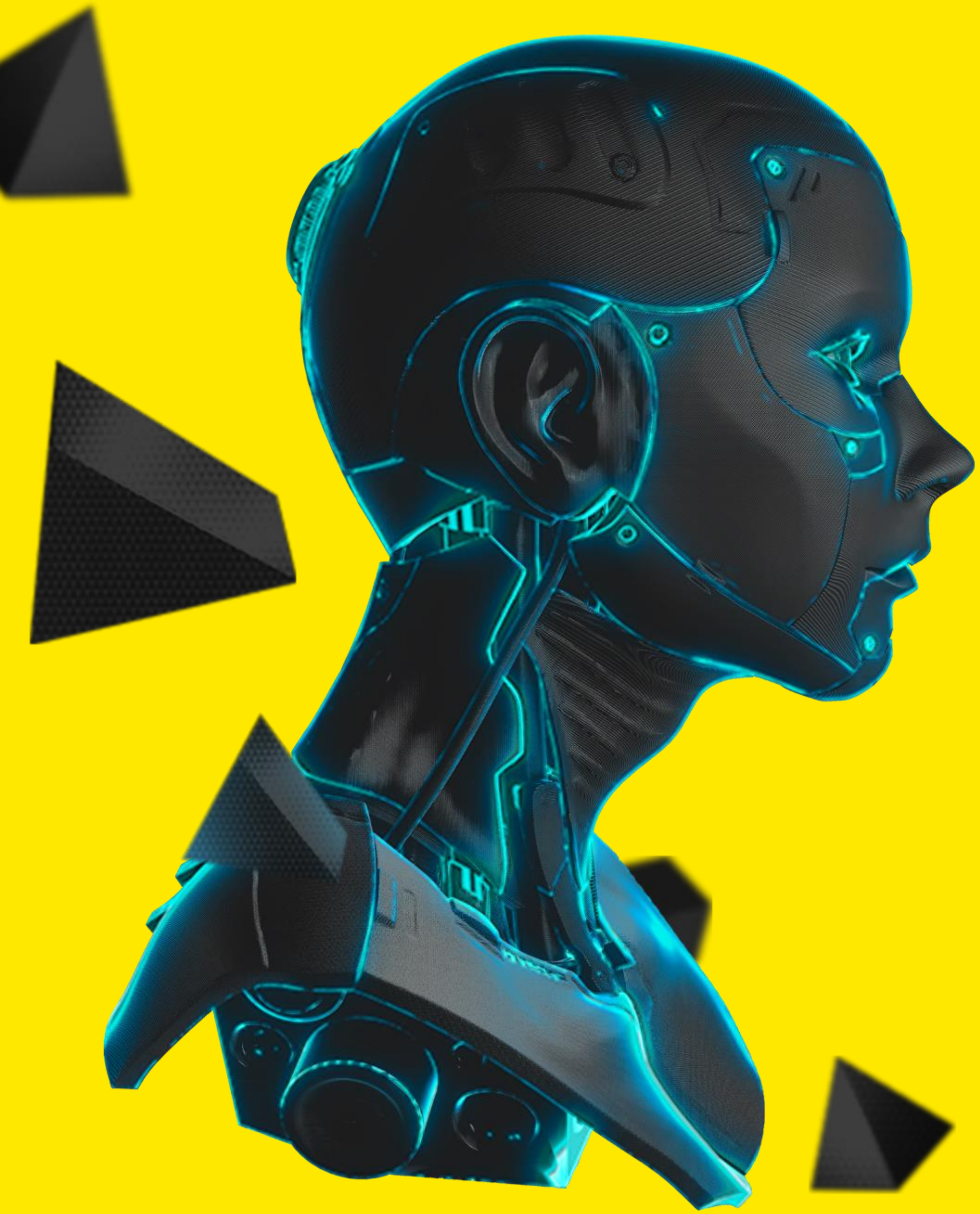

SIGGRAPH 2021

SOLID-FLUID INTERACTION WITH SURFACE-TENSION-DOMINANT CONTACT

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(* joint first authors)

Motivation


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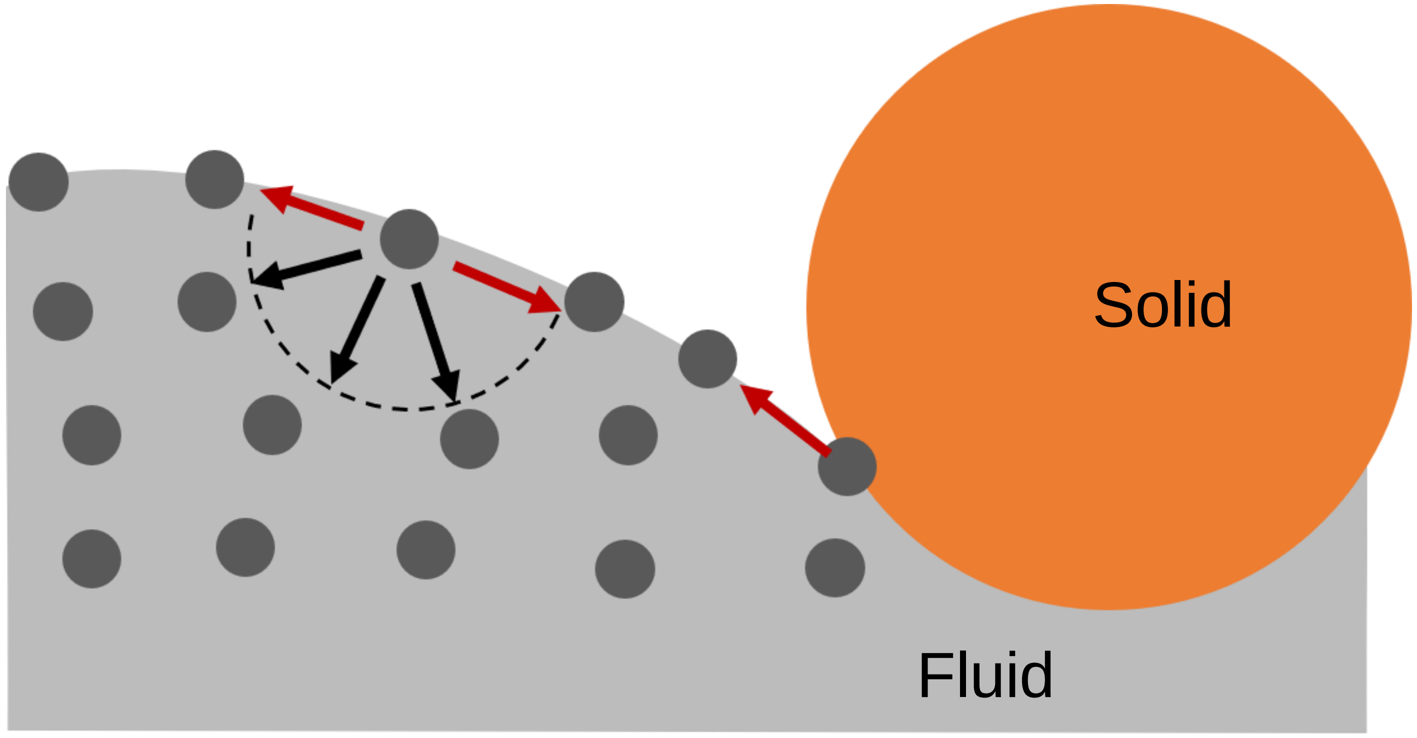
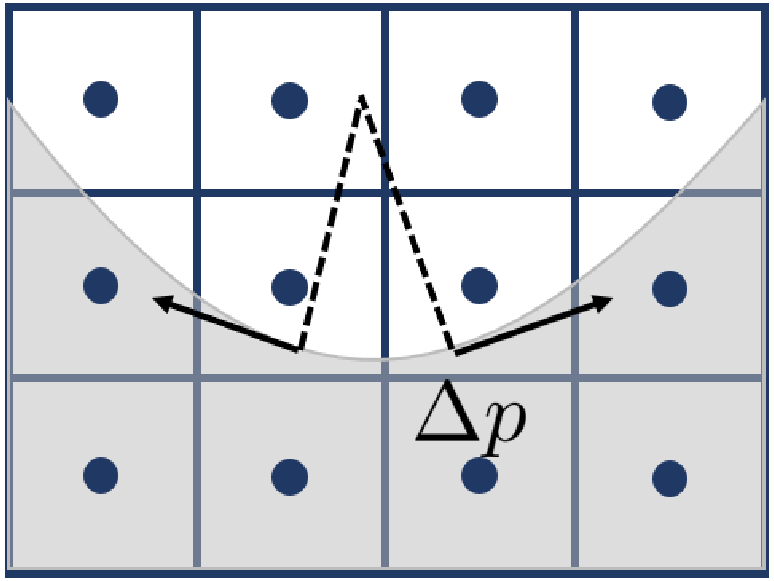
Motivation

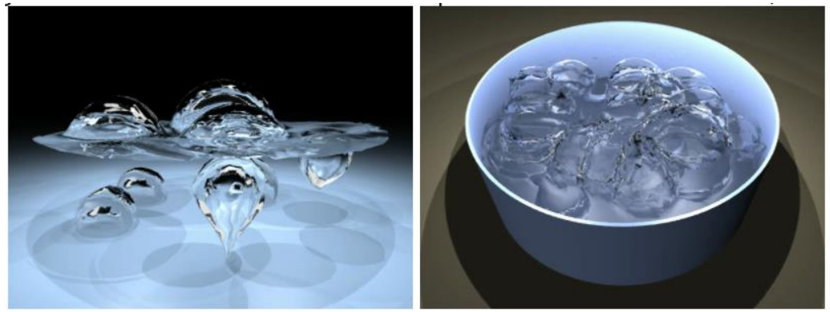


Surface tension

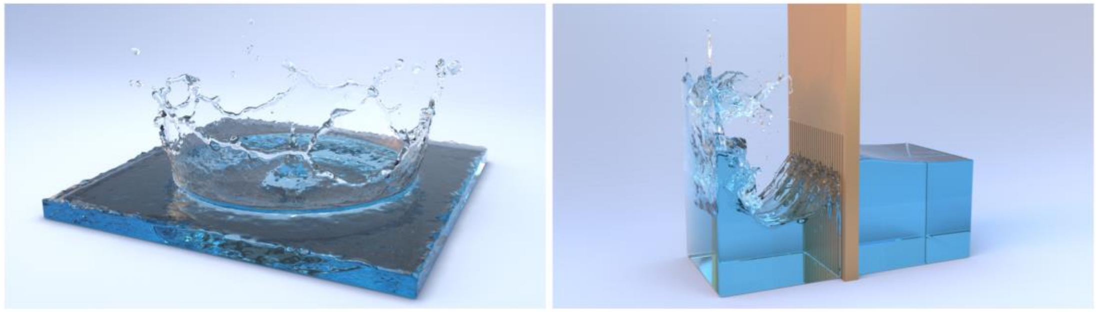
Fluid Simulation

- Eulerian method
  - Solve NS equation on MAC grid
- Implicit surface
  - Level set method
- Surface tension
  - Young-Laplace equation
  - Mean curvature



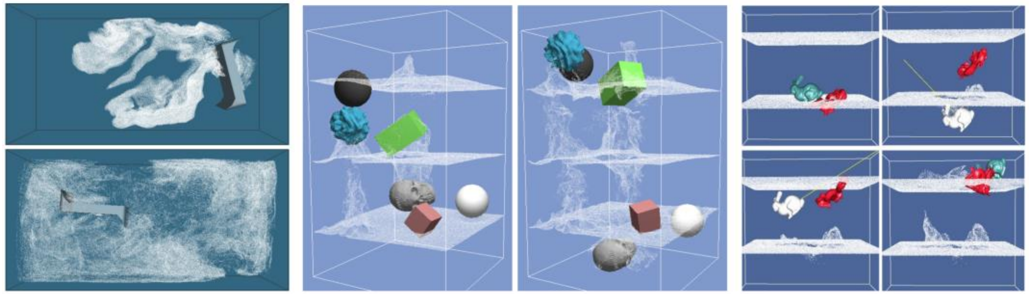


[Zheng et al. 2006]

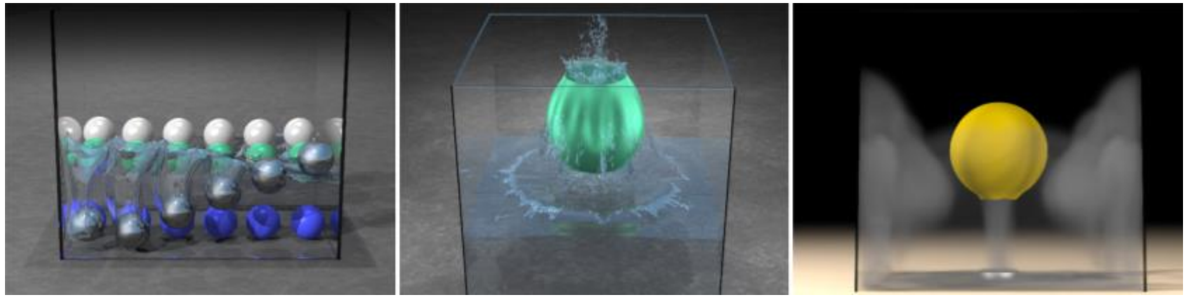


[Chen et al. 2020]

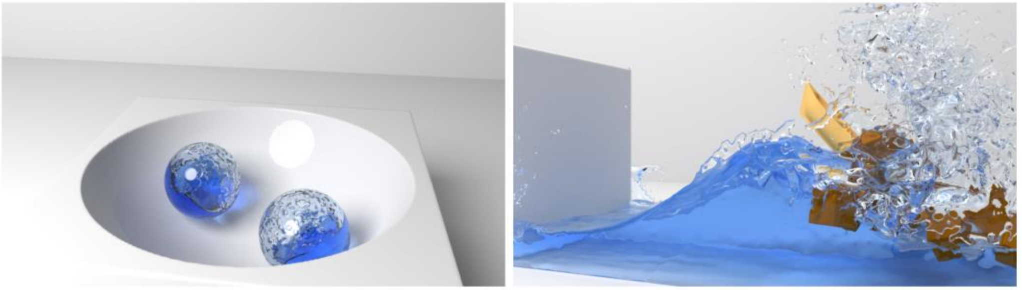
Fluid-Solid Coupling



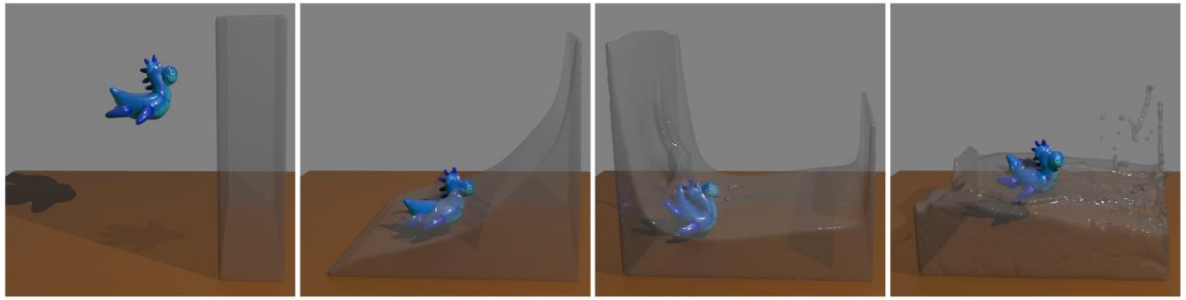
[Batty et al. 2007]



[Robinson-Mosher et al. 2008]



[Takahashi et al. 2020]



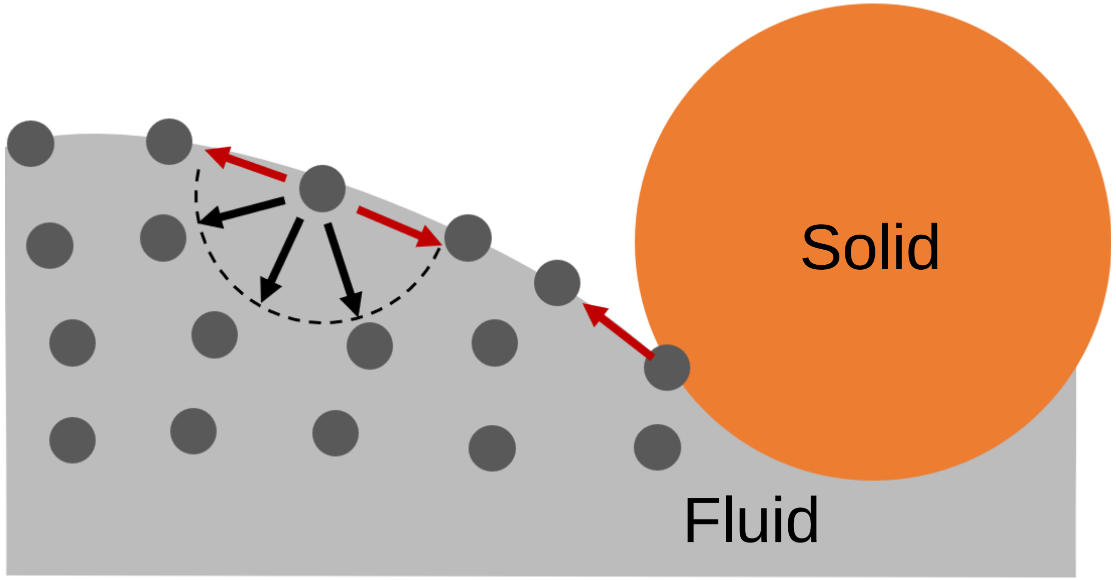
[Zarifi et al. 2017]

Eulerian fluid & Lagrangian solid

No surface tension

Problems

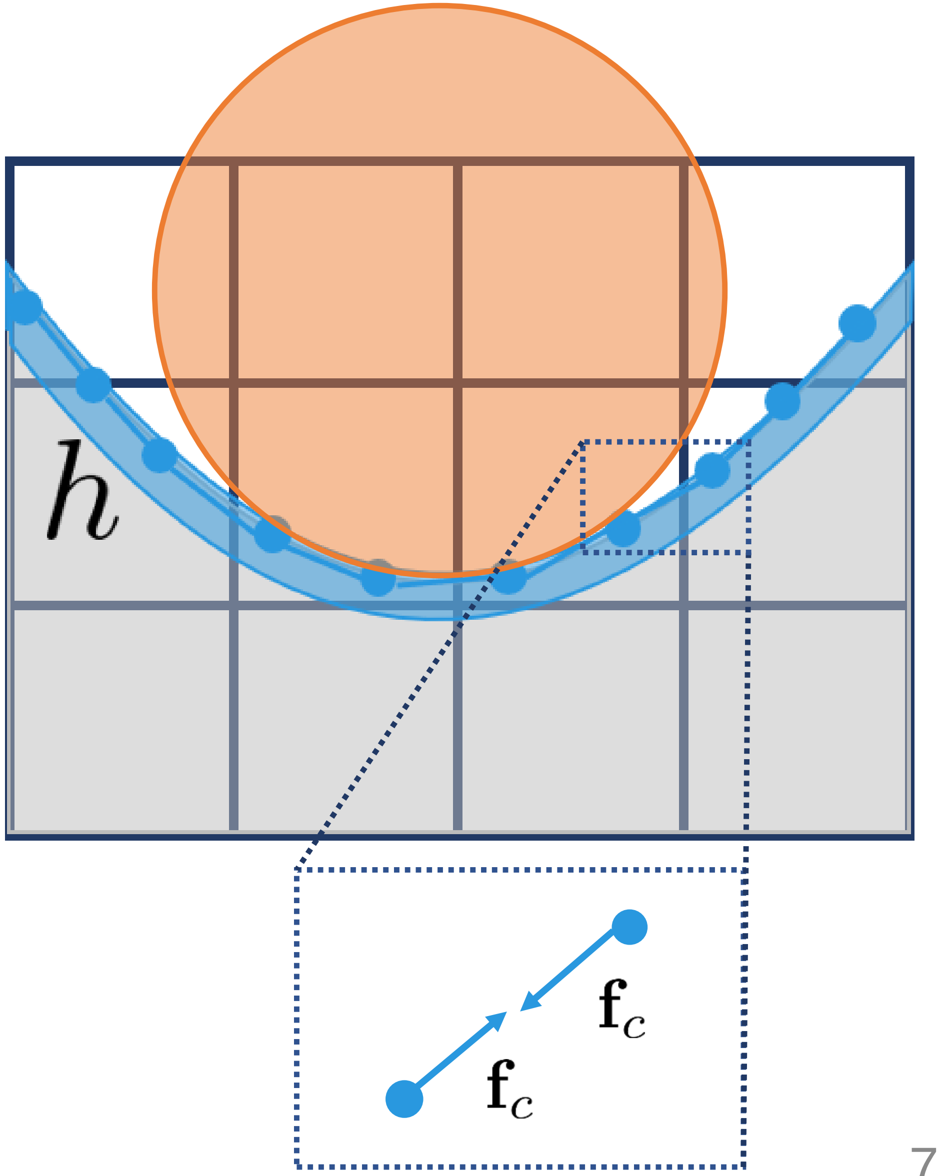
- Curvature’s precision
- Strong surface tension
- Apply surface tension to the solid



Use Lagrangian surface

Membrane

- Explicit mesh
  - Finer than the grid
- Mass
  - Finite thickness h
- Surface Tension
  - Nodal attraction
- Coupling
  - Momentum transfer



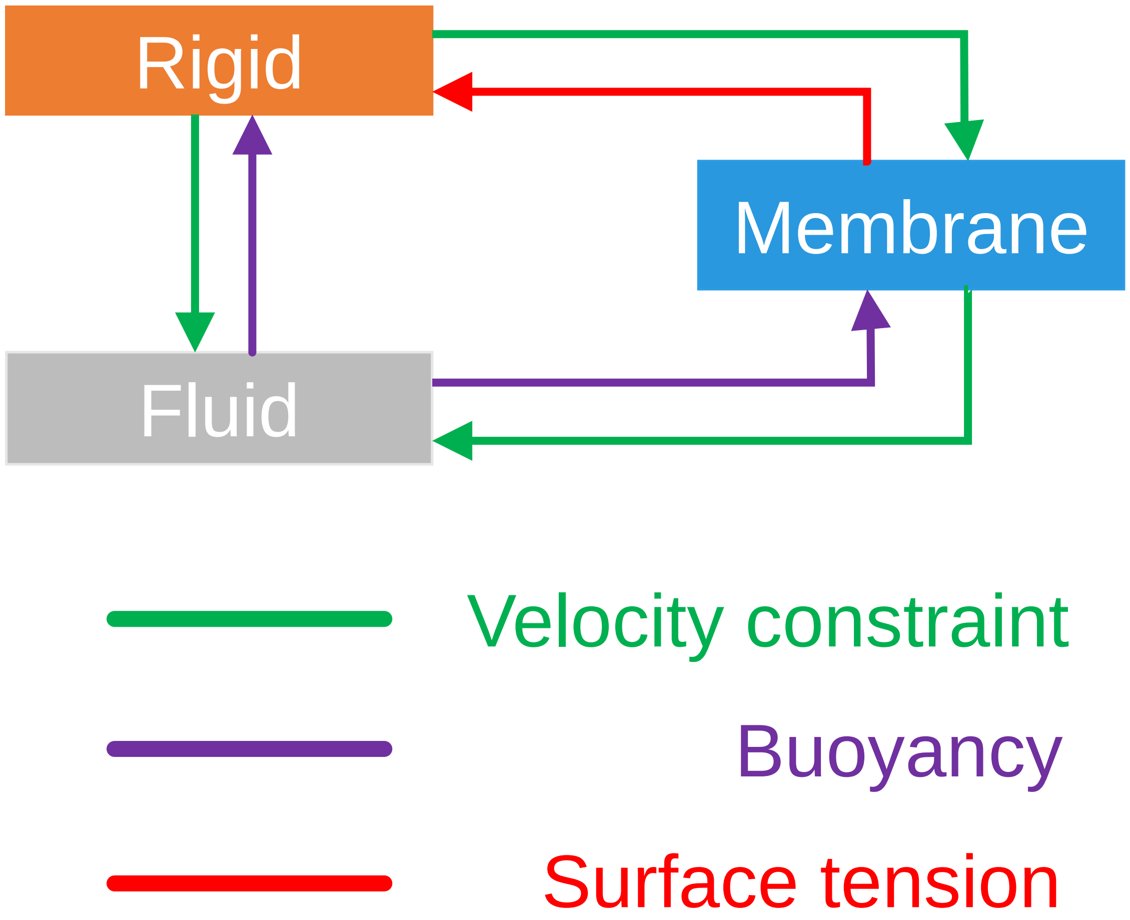
Real image
Simulation result
$\sigma = 72.8\ \mathrm{dyn/cm}$
$\rho_r = 7.9\ \mathrm{g/cm^3}$

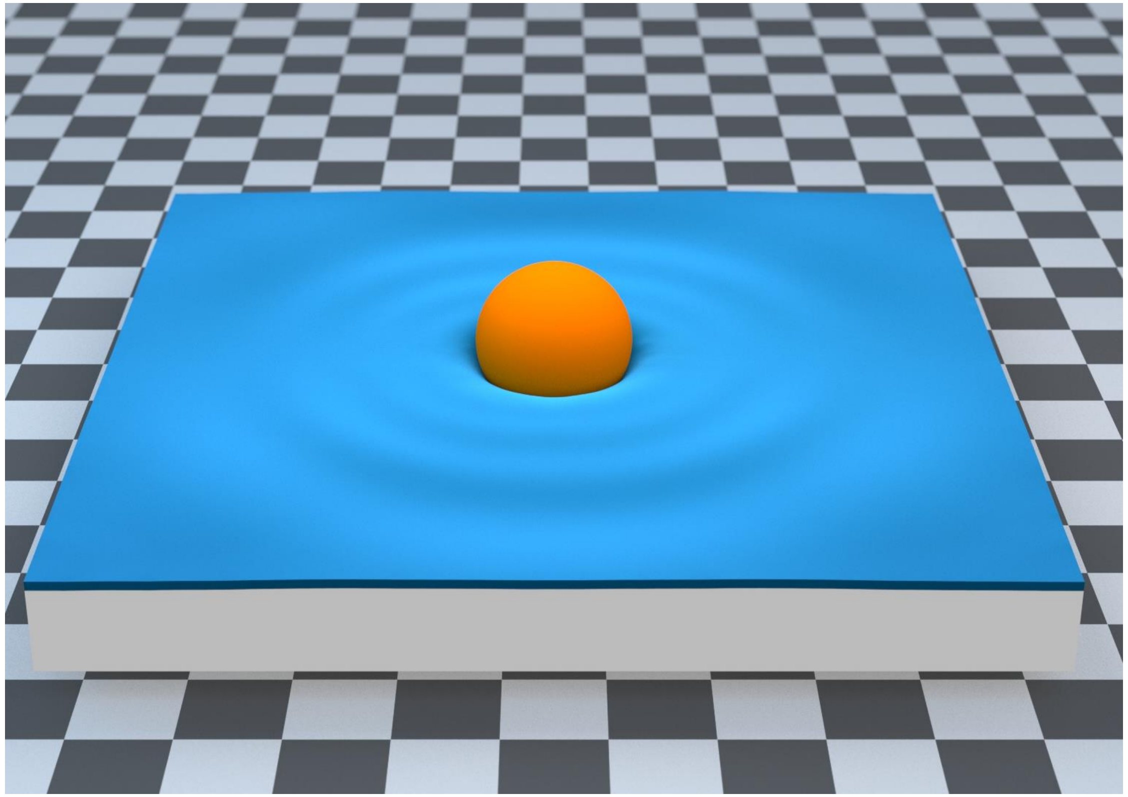
Our Contributions

- A novel Lagrangian thin membrane representation
- A monolithic coupling framework
- A prediction-correction contact handling scheme

Coupling System

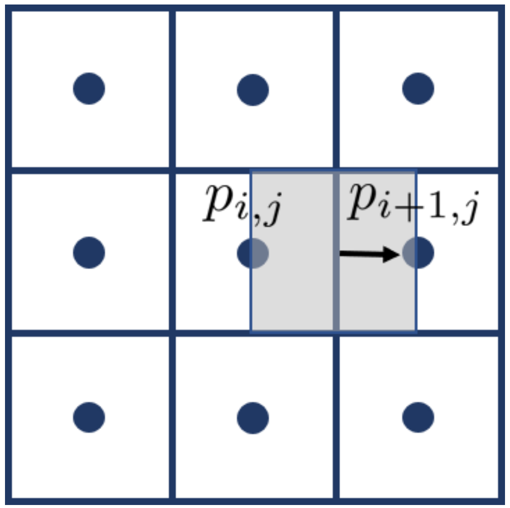
Three-way Coupling





Substance Modeling

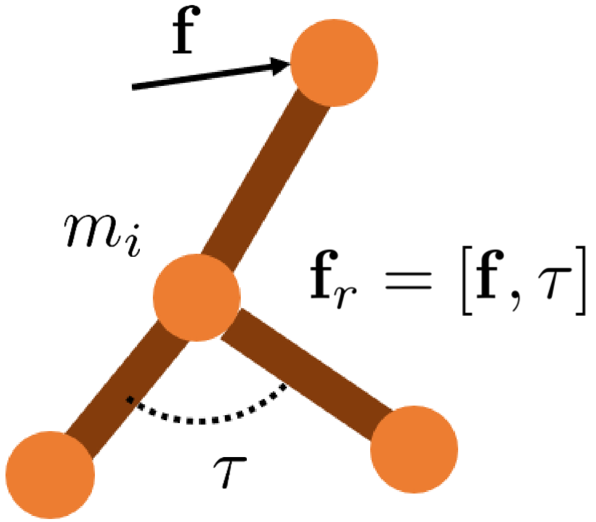
Fluid



Grid, Projection

$$\frac{1}{\rho}\mathbf{G}^T\mathbf{G}\mathbf{p}\Delta t = \mathbf{G}^T\mathbf{u}^*$$

Rigid



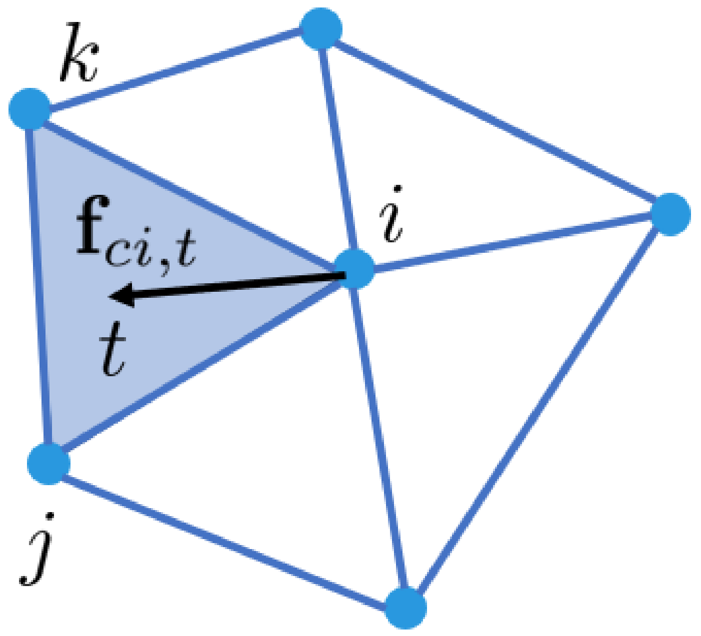
Lagrangian, Newton

$$\mathbf{M}_r\frac{\mathbf{v}_r^{n+1} - \mathbf{v}_r^n}{\Delta t} = \hat{\mathbf{f}}_r(\mathbf{q}_r^{n+1}, \mathbf{v}_r^n)\Delta t$$

Substance Modeling

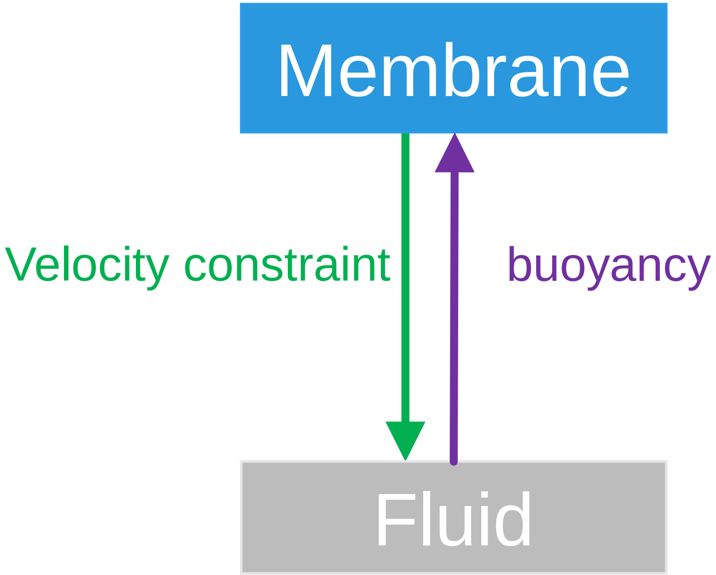
- Surface energy: $E = \sigma A$
  - σ: surface tension coefficient
  - A: surface area
- Nodal force: $\mathbf{f}_{ci,t} = -\sigma \frac{\partial A_t}{\partial \mathbf{x}_i}$
- Mass: $m_s = \rho h A$
  - ρ: density of the fluid
  - h: membrane thickness
- Newton equation: $\mathbf{M}_s \frac{\mathbf{v}_s^{n+1} - \mathbf{v}_s^n}{\Delta t} = \mathbf{f}_c$

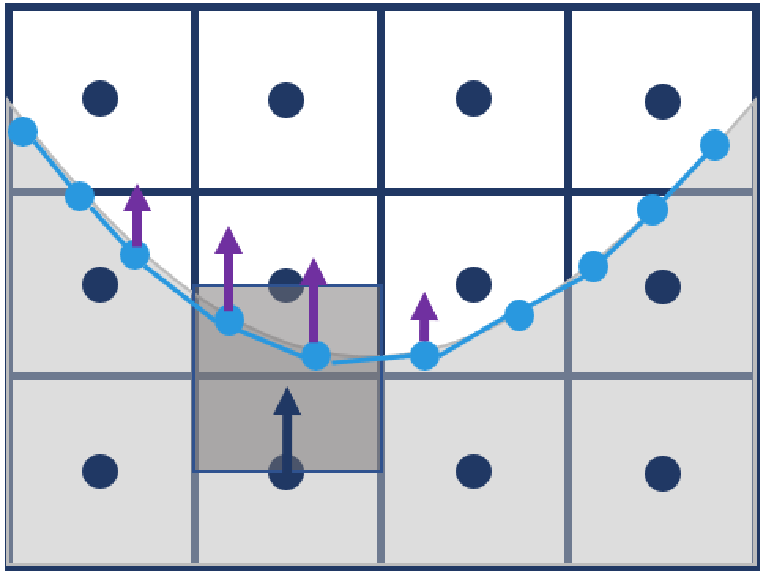
Membrane

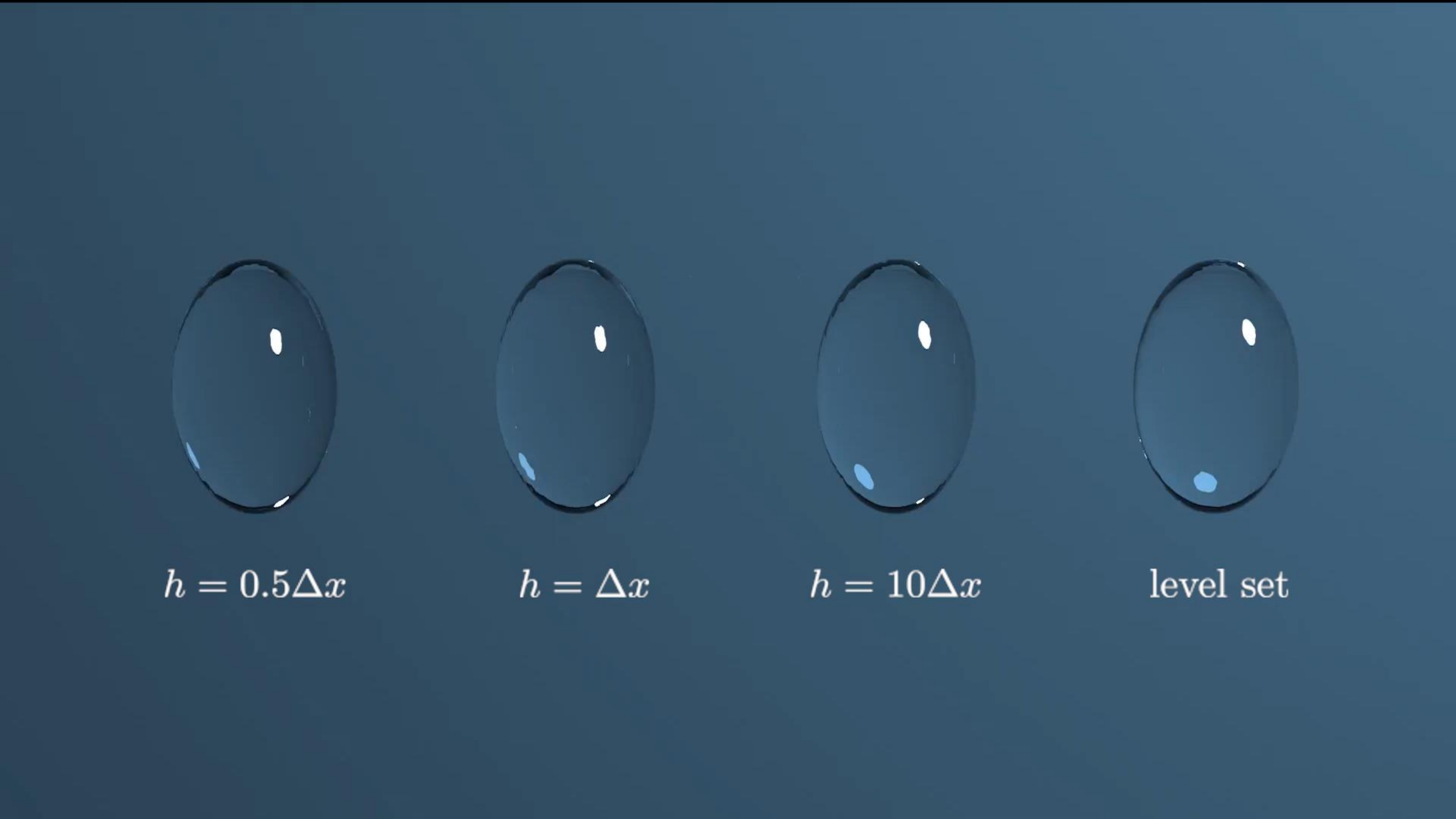


Fluid-Membrane

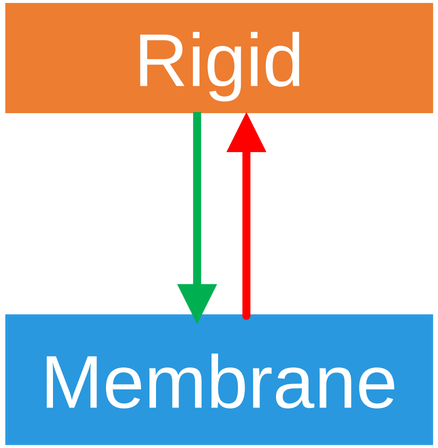
$$\begin{bmatrix} \frac{V}{\rho}\mathbf{G}^T\mathbf{G} & -V\mathbf{G}^T\mathbf{W} \\ -\mathbf{W}^T\mathbf{G}V & -\hat{\mathbf{M}}_s \end{bmatrix} \begin{bmatrix}\hat{\mathbf{p}} \\ \mathbf{v}_s^{n+1}\end{bmatrix} = \begin{bmatrix} V\mathbf{G}^T\mathbf{u}^* \\ -\mathbf{M}_s\mathbf{v}_s^n - \mathbf{W}^T\mathbf{M}\mathbf{u}^* - \hat{\mathbf{f}}_c\Delta t\end{bmatrix}$$



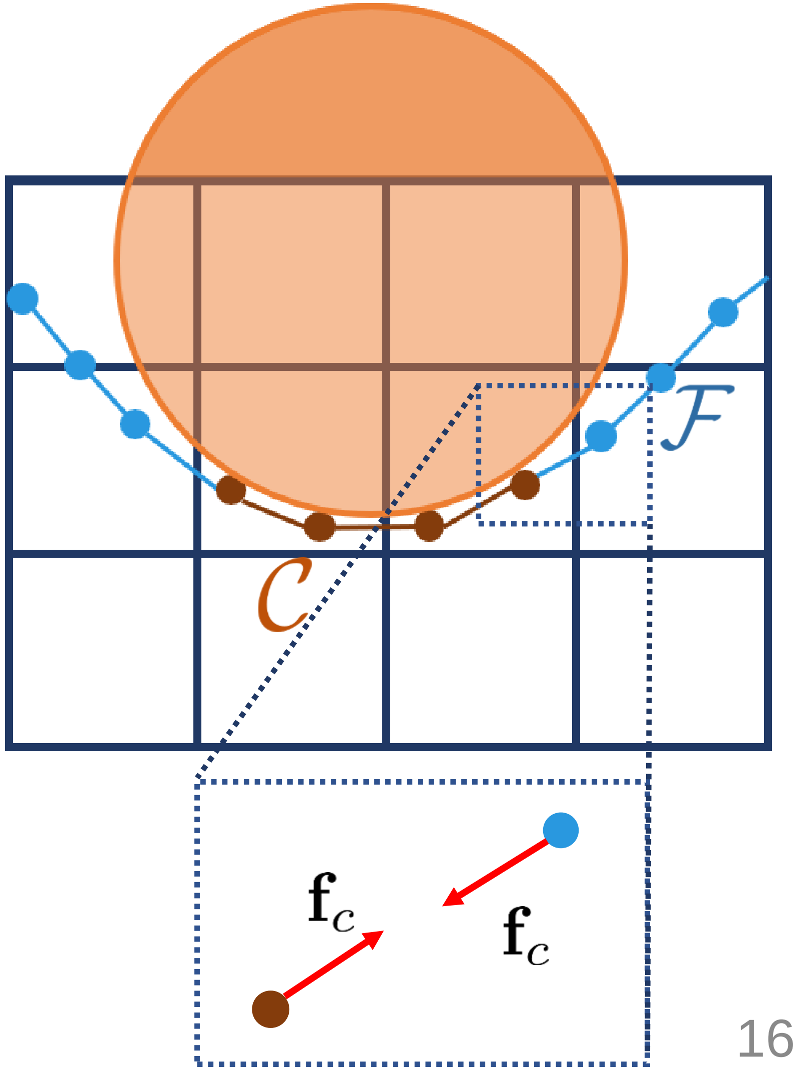



$h = 0.5\Delta x$
$h = \Delta x$
$h = 10\Delta x$
level set

Membrane-Rigid

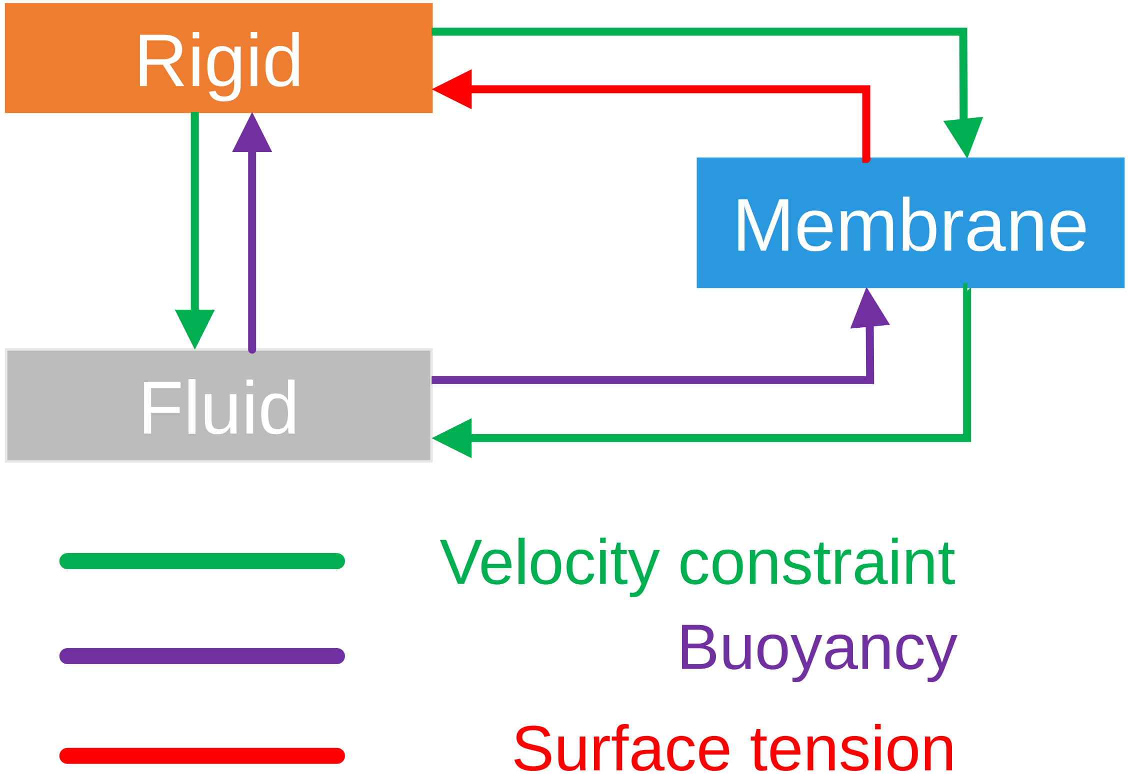


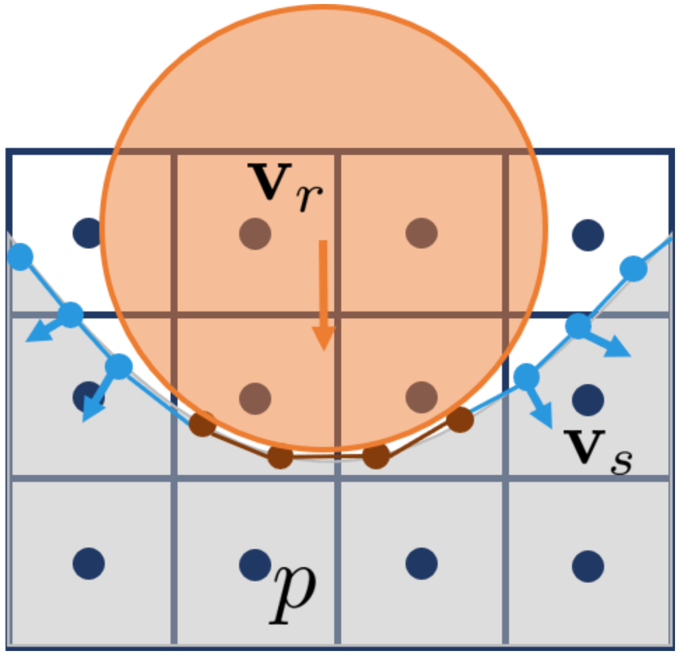
- Divide membrane into C and F
- Velocity constraint: for C
- Surface tension: between C and F



Three-way Coupling System

$$\begin{bmatrix} \frac{V}{\rho}\mathbf{G}^T\mathbf{G} & -V\mathbf{G}^T\mathbf{W} & -V\mathbf{G}^T\mathbf{J}_r \\ -\mathbf{W}^T\mathbf{G}V & -\tilde{\mathbf{M}}_s & -\mathbf{K}_{c,r}\Delta t^2 \\ -\mathbf{J}_r^T\mathbf{G}V & -\mathbf{K}_{a,s}\Delta t^2 & -\tilde{\mathbf{M}}_r \end{bmatrix} \begin{bmatrix} \hat{\mathbf{p}} \\ \mathbf{v}_s^{n+1} \\ \mathbf{v}_r^{n+1} \end{bmatrix} = \begin{bmatrix} V\mathbf{G}^T\mathbf{u}^* \\ -\mathbf{M}_s\mathbf{v}_s^n - \tilde{\mathbf{f}}_c\Delta t - \mathbf{W}^T\mathbf{M}\mathbf{u}^* \\ -\mathbf{M}_r\mathbf{v}_r^n - \tilde{\mathbf{f}}_a\Delta t - \tilde{\mathbf{f}}_r\Delta t - \mathbf{J}_r^T\mathbf{M}\mathbf{u}^* \end{bmatrix}$$



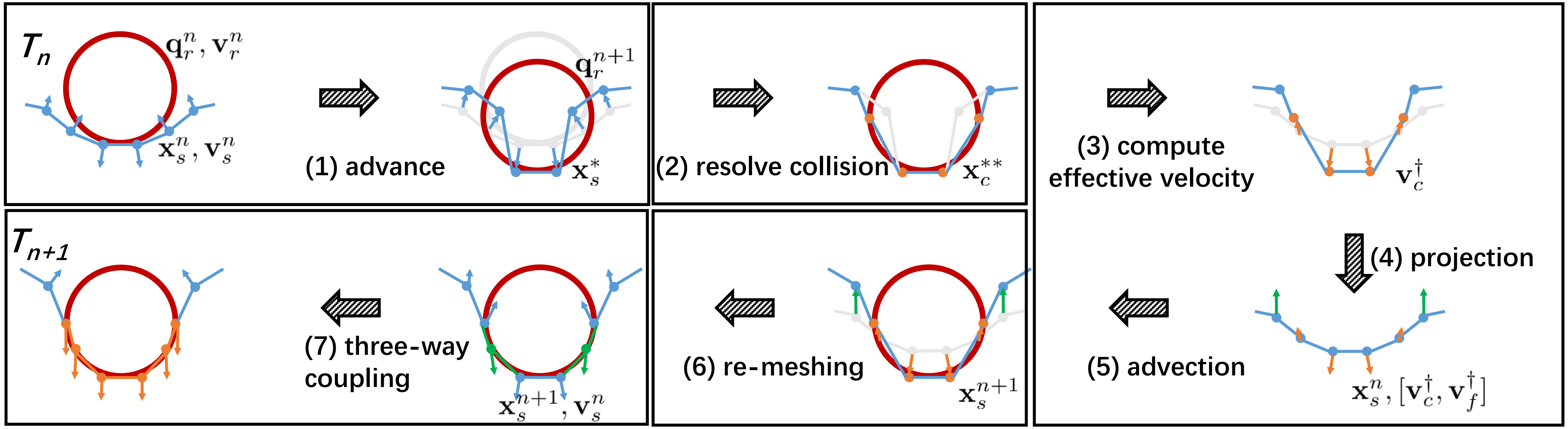


Time Scheme

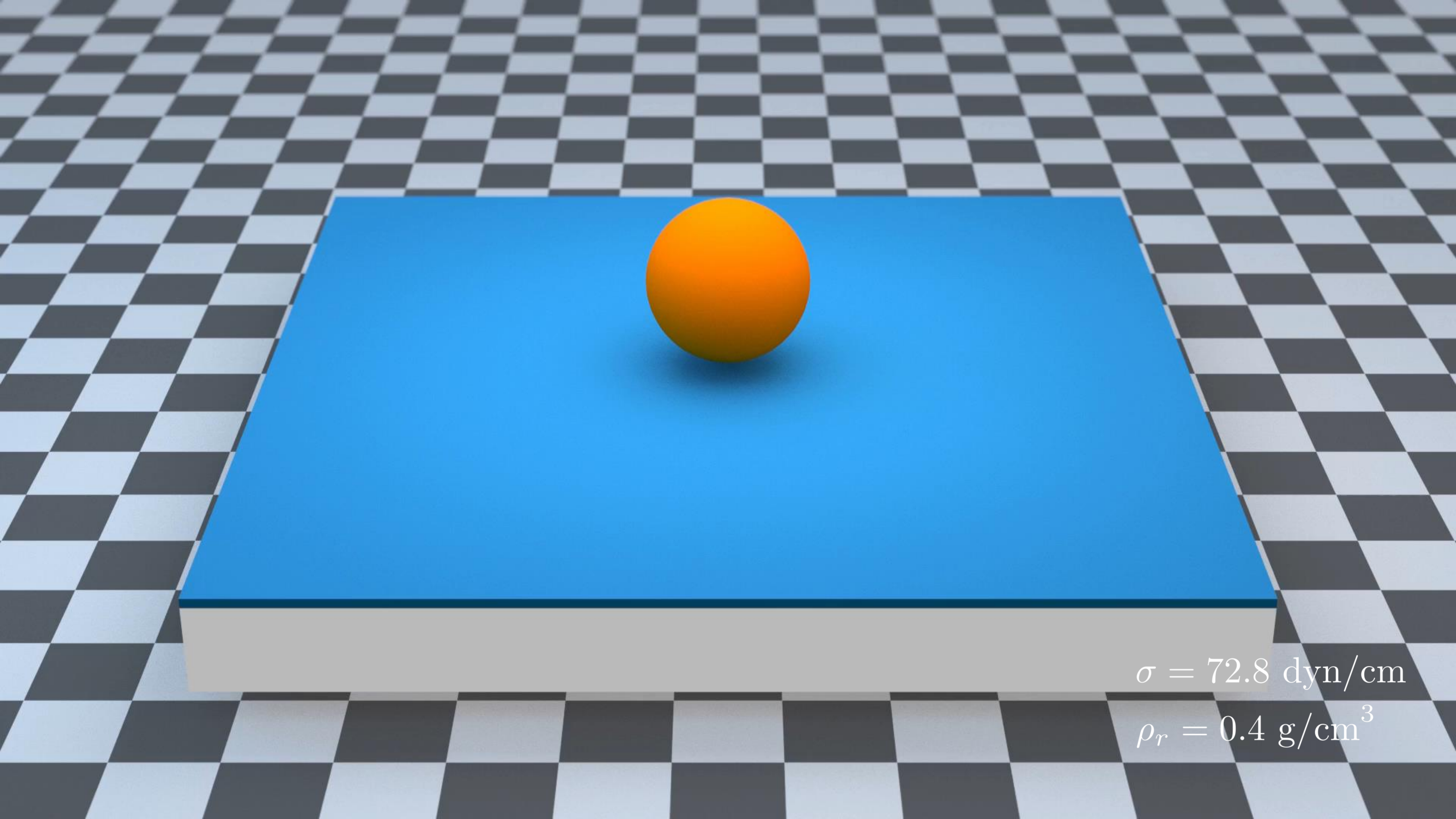
- Advance position
- Resolve collision
- Correct fluid volume: first solve
- Re-meshing
- Three-way coupling: second solve

Prediction

Correction

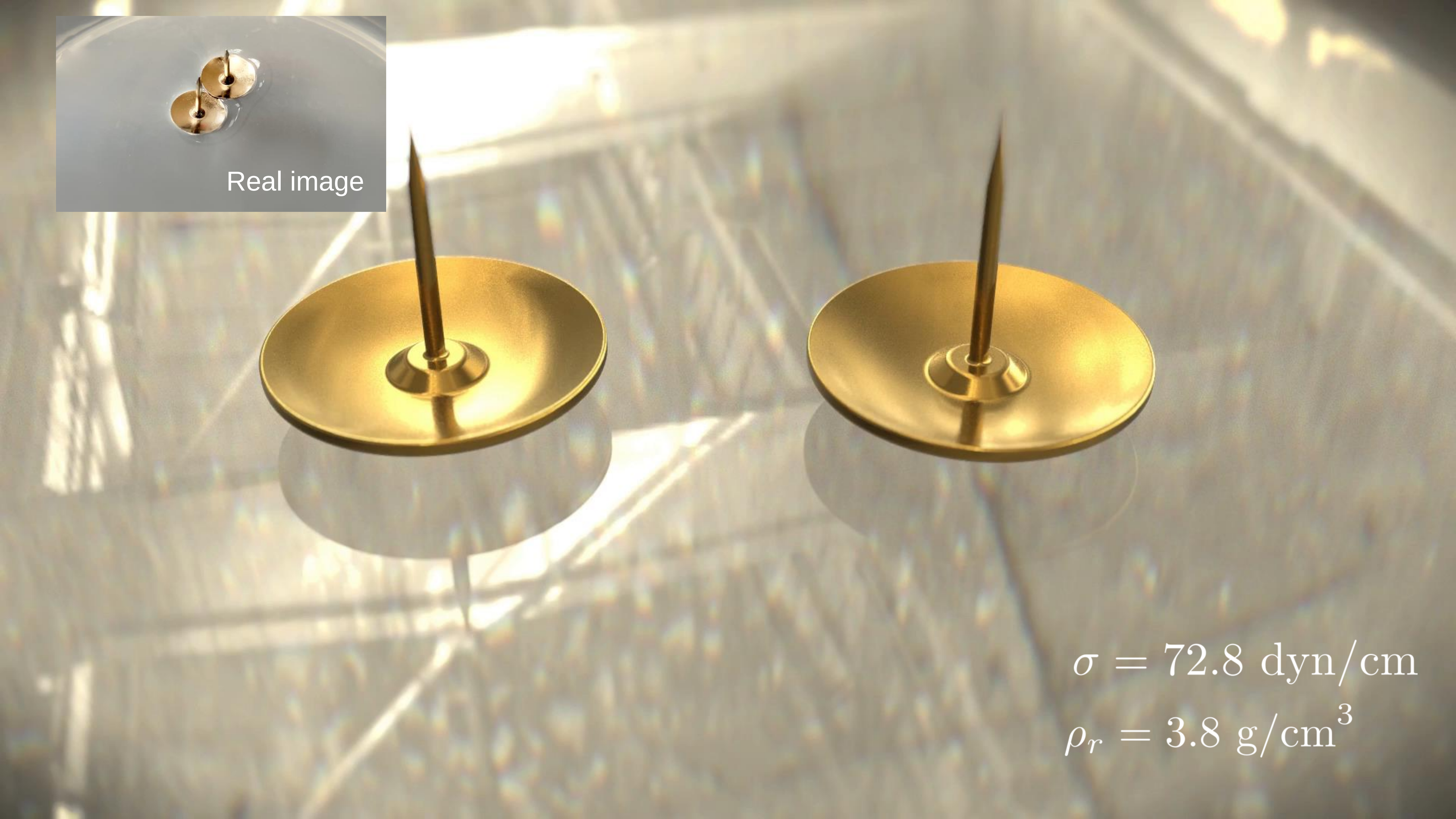


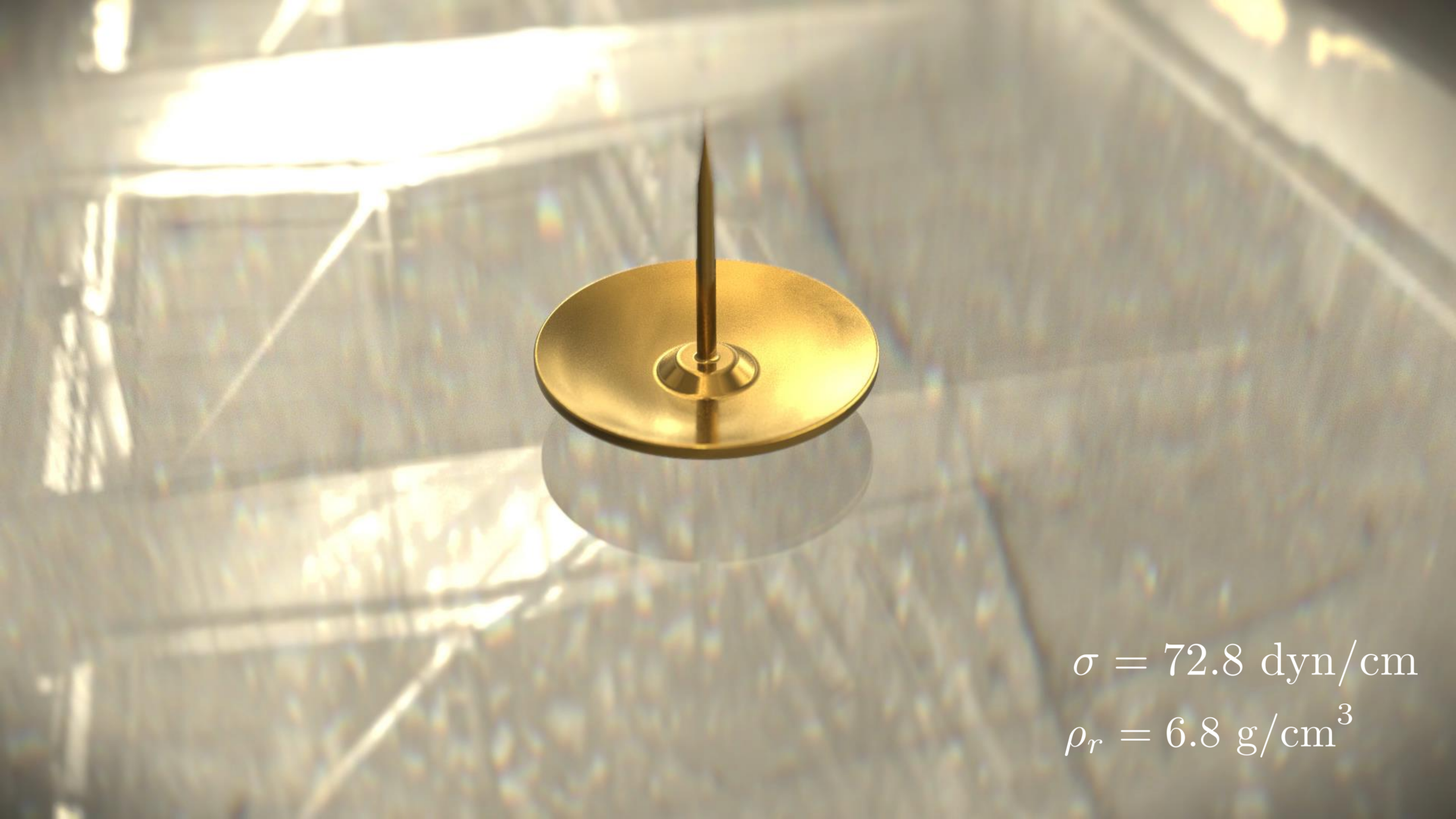
Simulation Results


$\sigma = 72.8$ dyn/cm
$\rho_r = 0.4$ g/cm^3

$$\sigma = 72.8 \text{ dyn/cm}$$

$$\sigma = 46 \text{ dyn/cm}$$

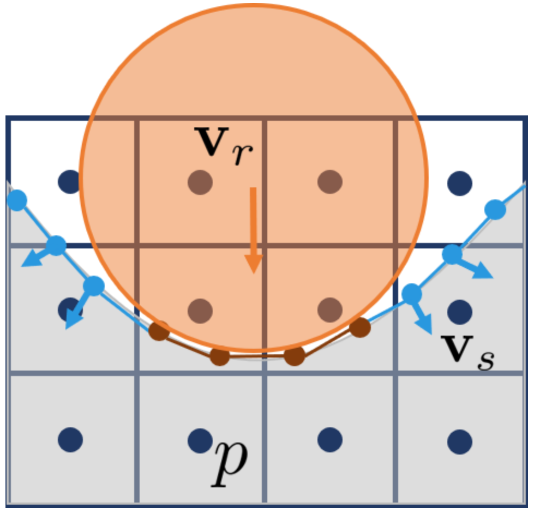

Real image
$\sigma = 72.8\ \mathrm{dyn/cm}$
$\rho_r = 3.8\ \mathrm{g/cm^3}$


$\sigma = 72.8$ dyn/cm
$\rho_r = 6.8$ g/cm^3

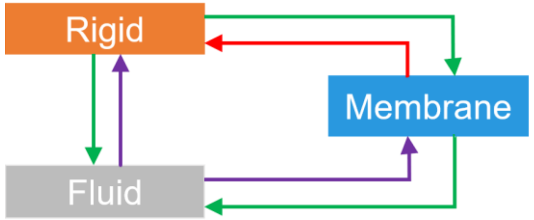

Real image
$\sigma = 72.8$ dyn/cm
$\rho_r = 0.1$ g/cm^2


$\sigma = 72.8$ dyn/cm
$\rho_r = 0.4$ g/cm^3

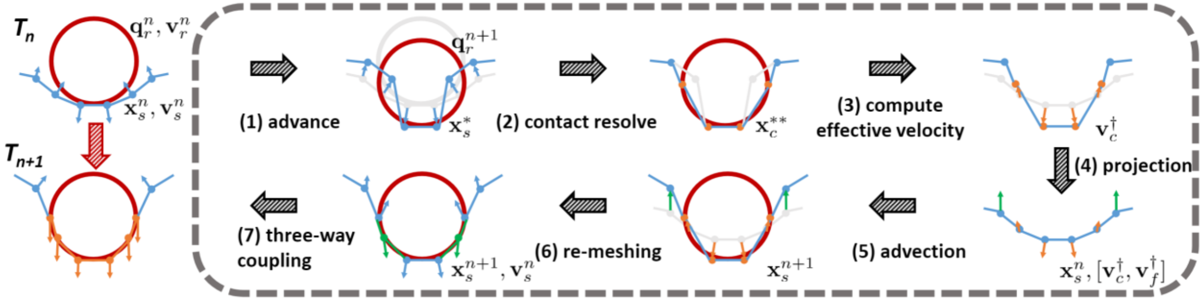
Summary



- Surface-tension-dominant solid-fluid coupling
  - Membrane representation
  - Three-way coupling
  - Prediction-correction time scheme



- Limitations
  - Large topology change
  - Contact angle
  - Efficiency



Thanks!